SABES Math Bulletin

Building Research Into Practice
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In This Issue

Fall means new programs starting and often it means meeting new students. Because this is the traditional 'start of the school year,' we are focusing on some basic principles: what characterizes effective instructors, and what strategies research shows are effective when teaching mathematics. We hope these primers help you start off your year on firm footing.

Also in this edition, we share some recent research about brain hardwiring for math, as well as some information about the 'levels of knowing' in mathematics, which can help direct us when designing activities and lessons for students at all levels of understanding.

Readers interested in creating a classroom culture that fosters equity and mathematical growth will want to check out "How Did You Solve That Problem" on page 6.

Together, we hope these research pieces help you become a stronger teacher and help math class become the best part of each student's time at the learning center.

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NCTM's Effective Strategies Brief

Effective Strategies for Teaching Students with Difficulties in Mathematics

The material below is excerpted from the NCTM *Effective Strategies Brief,* available on line at: <http://nctm.org/news/content.aspx?id=8452>

"This research brief focuses on evidence-based practices for teaching students with difficulties in mathematics. Most of the summary for this research brief is based on two recently conducted meta-analyses (Baker, Gersten, and Lee 2002; Gersten et al. 2006) as well as complementary work by Kroesbergen and van Luitt (2003). Together, the reviews encompass more than fifty studies, and although this is an emerging and substantial research base, it is far from definitive. As a composite, the studies reviewed present a picture of specific aspects of instruction that are consistently effective in teaching students who experience difficulties with mathematics. The principles that emerged from the research seem appropriate for instruction in a variety of situations and possible settings.

Six aspects of instruction have been studied in depth. Table 1on page 3 lists each of these along with the average effect size for teaching special education students (Gersten et al. 2006) and other students with difficulties learning mathematics (Baker, Gersten, and Lee 2002). Effect sizes of 0.2 are considered small, 0.4 moderate, and 0.6 or above large. A small effect might raise students' scores on a standardized test about 8 percentile points; a large effect would raise a score approximately 25 percentile

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Effective Strategies . . .
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Systematic and Explicit Instruction

Consistently strong effects were found for systematic, explicit instruction. We (NCTM) define explicit instruction as instruction that involves a teacher demonstrating a specific plan (strategy) for solving the problem types and students using this plan to think their way through a solution. In most studies, the emphasis was placed on providing highly explicit models of steps and procedures or questions to ask in solving problems. The degree of structure and specificity is atypical in conventional mathematics texts.

We divided the explicit instruction studies into two categories: those involving only one problem type, and those involving multiple problem types. In both instances, mean effect sizes were large for both the special education students and the population of low-performing students with no specific learning disability. Although the majority of studies dealt with procedural knowledge, many students with learning disabilities in mathematics struggle with what are considered basic mathematical procedures. This, in turn, limits their ability to solve more complex problem types in which basic procedures are embedded.

Student Think-Alouds

Studies showed that when faced with multi-step problems, students frequently attempted to solve the problems by randomly combining numbers instead of implementing a solution strategy step by step. The process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem— was consistently effective. In part, this procedure may be effective because the impulsive approach to solving problems taken by many students with mathematics difficulties was addressed. Results of these students were quite impressive, with an average effect size of 0.98, which is very large. …

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Conclusion

In summary, the relatively small body of instructional research suggests several important teaching practices. For low-achieving students, the use of structured peer-assisted learning activities, along with systematic and explicit instruction and formative data furnished both to the teacher and to the students, appears to be most important. For special education students, explicit, systematic instruction that involves extensive use of visual representations appears to be crucial. In many situations with special education students, it is often advantageous for students to be encouraged to think aloud while they work, perhaps by sharing their thinking with a peer. These approaches also seem to inhibit those students who try too quickly and impulsively to solve problems without devoting adequate attention to thinking about what mathematical concepts and principles are required for the solution. Instruction should ideally be in a small group of no more than six and (a) address skills that are necessary for the unit at hand, (b) be quite explicit and systematic, and (c) require the student to think aloud as she or he solves problems or uses graphic representation to work through problem-solving options. Finally, it should balance work on basic whole-number or rational-number operations (depending on grade level) with strategies for solving problems that are more complex. These criteria should be considered in evaluating intervention programs for working with these types of students."

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TABLE 1: Effect Sizes for Instructional Variables for Special Education Students and Other Low-Achieving Students

Instructional Strategy	Effect Size for Special Education Students	Effect Size for Low-Achieving Students
1. Visual and graphic descriptions of problems	0.50 (moderate)	N/A
2. Systematic and explicit instruction	1.19 (large)	0.58 (moderate to large)
3. Student think-alouds	0.98 (large)	N/A
4. Use of structured peer-assisted learning activities involving heterogeneous ability groupings	0.42 (moderate)	0.62 (large)
5. Formative assessment data provided to teachers	0.32 (small to moderate)	0.51 (moderate)

Describing Effective Instructors

Editor's Note: *Material quoted in the following article is taken primarily from "Campbell, Pat. (2005) Hardwired for hope: Effective ABE/Literacy Instructors," Literacies: Researching practice; practicing research, Canada, #5, Spring. The journal can be subscribed to and accessed through the Literacies website: http://www.literacyjournal.ca/*

Effective instructors are passionate about their work, exercise strong emotional intelligence skills, reflect on their teaching, and put their students first. According to Pat Campbell, who reviewed the 2004 book Hardwired for Hope: Effective ABE/Literacy Instructors, "…the data supporting these conclusions came from two primary sources: the practitioner/research group (who authored the book) and 17 instructors" (Campbell, p.43) who were interviewed by the researchers.

Campbell explains that the Hardwired for Hope authors devote three chapters to their findings. One chapter discusses the characteristics of effective instructors, a second focuses on the motivation and beliefs of effective instructors, and a third centers on the styles, strategies and skills of effective instructors.

What did the researchers discover? "This study challenges the assumption that effective instructors are those who have a firm grasp on content or subject matter. Rather, there were clear indications from the data 'of the importance of the influence of emotions in effective instruction' (p. 56)." (Campbell, p.44) Because of the many differences among ABE students and between ABE students and instructors, effective instructors must "know the importance of working across differences in the classroom in order to create a learning environment that is safe, supportive and provides a feeling of comfort." (Idem.)The belief system that drives or motivates the work of effective instructors is conceived as containing five convictions:

- ABE/literacy students are powerful, self-determined adults with the right to make their own decisions.
- Making a connection with students is a necessity, a joy and a challenge.
- We do not blame students for the effects larger societal forces have made and are still making on them.
- A positive learning experience is essential for student success and usually must be accomplished

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in the face of residual anger, resentment and fear bout schooling.

- Instructors can make a difference in the quality of students' lives and communities.

Challenging Maslow

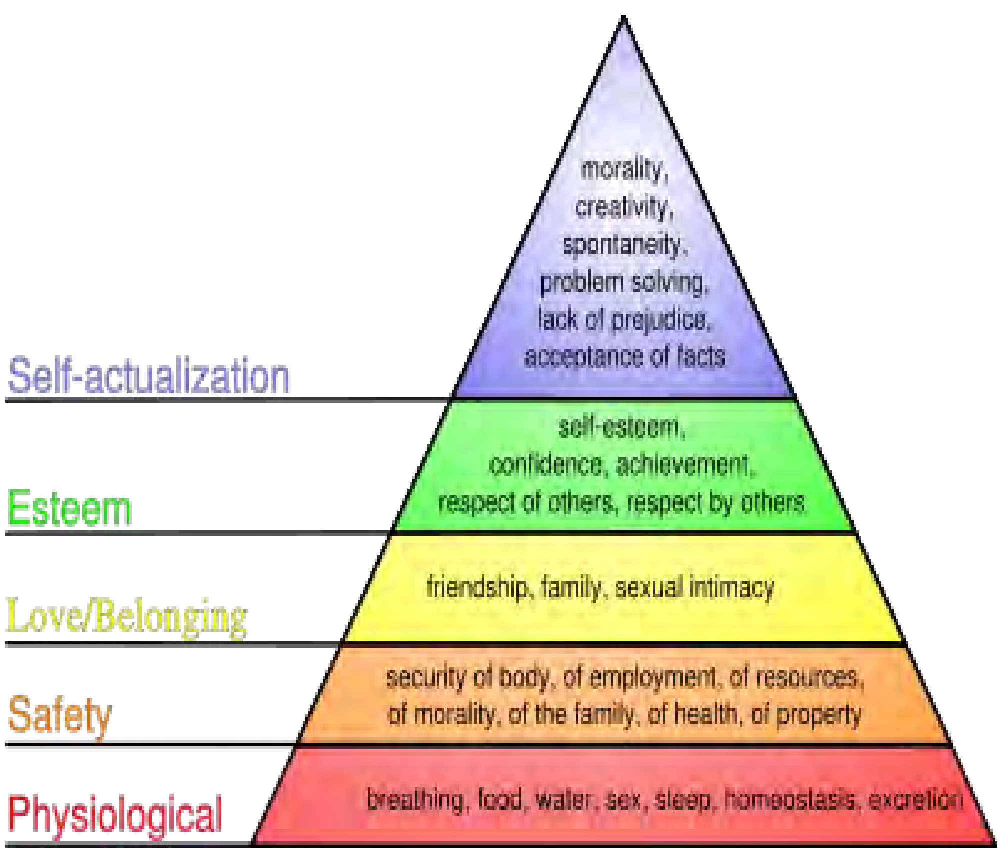
Campbell declares that "When these convictions are played out, they challenge some commonly held assumptions. For instance, the assumption that people need safety, food and shelter before they can be motivated to learn and grown is challenged. Lucy Alderson, an instructor who works with women in the sex-trade states, '…We've just thrown out Maslow's hierarchy (p.108). Alderson views literacy as a right that needs to be pursued, no matter where people are on Maslow's hierarchy of needs." (Idem.)

As for the styles, strategies and skills employed by effective instructors, they have in common a "focus on the needs of the individual," the creation of "a physical and emotional environment that promotes learning, trust, sharing, dialogue and growth," and a propensity to reflection "on their delivery styles, teaching philosophy, classroom interaction, student feedback, and their role as facilitator." (Campbell, p. 45) As the authors pored through their research data, they discovered that specific techniques and strategies were not detailed, rather effective instructors "preferred to talk about the underlying reflections that led to specific strategies." (Idem.)

First and foremost, however, effective instructors respond to their students as individuals and collectively. This commitment to students, Campbell concludes, "is reflected in the closing sentence of Hardwired for Hope:

Balancing our jobs in ways that keep us emotionally and physically healthy is vital and, as many of struggle to do this, we still choose to put our students first and foremost (Hardwired for Hope, p. 171).

Maslow's Heirarchy of Needs



Levels of Knowing Math

As you present your lessons and assess student learning, it may be helpful to recall the levels of knowing mathematics presented by Mahesh, Sharma, former director of the Center for Teaching/Learning Mathematics in Framingham, MA. In 1990, Sharma outlined six "levels of knowing math." He recommended proceeding through the levels when instructing students who have not mastered particular math content. The levels are:

- Intuitive
- Concrete
- Pictorial
- Abstract
- Application Communication

Locating the Math Instinct

In a September 15 New York Times article, "Gut Instinct's Surprising Role in Math," author Natalie Angier reported recent research that indicates humans may be hardwired for comparing quantities at a glance. The ability to compare numbers of differently colored dots appears to be correlated with (though not causative of) individuals' propensity for math. Read the excerpt below to find out more about this amazing research discovery.

"…This month in the journal Nature, Justin Halberda and Lisa Feigenson of Johns Hopkins University and Michele Mazzocco of the Kennedy Krieger Institute in Baltimore described their study of 64 14-year-olds who were tested at length on the discriminating power of their approximate number sense. The teenagers sat at a computer as a series of slides with varying numbers of yellow and blue dots flashed on a screen for 200 milliseconds each — barely as long as an eye blink. After each slide, the students pressed a button indicating whether they thought there had been more yellow dots or blue.)

Given the antiquity and ubiquity of the nonverbal number sense, the researchers were impressed by how widely it varied in acuity. There were kids with fine powers of discrimination, able to distinguish ratios on the order of 9 blue dots for every 10 yellows, Dr. Feigenson said. "Others performed at a level comparable to a 9-month-old," barely able to tell if five yellows outgunned three blues. Comparing the acuity scores with other test results that Dr. Mazzocco had collected from the students over the past 10 years, the researchers found a robust correlation between dot-spotting prowess at age 14 and strong performance on a raft of standardized math tests from kindergarten onward. "We can't draw causal arrows one way or another," Dr. Feigenson said, "but your evolutionarily endowed sense of approximation is related to how good you are at formal math."

The researchers caution that they have no idea yet how the two number systems interact. Brain imaging studies have traced the approximate number sense to a specific neural structure called the intraparietal sulcus, which also helps assess features like an object's magnitude and distance. Symbolic math, by contrast, operates along a more widely distributed circuitry, activating many of the prefrontal regions of the brain that we associate with being human. Somewhere, local and global must be hooked up to a party line.

Other open questions include how malleable our inborn number sense may be, whether it can be improved with training, and whether those improvements would pay off in a greater appetite and aptitude for math. If children start training with the flashing dot game at age 4, will they be supernumerate by middle school?

Dr. Halberda, who happens to be Dr. Feigenson's spouse, relishes the work's philosophical implications. "What's interesting and surprising in our results is that the same system we spend years trying to acquire in school, and that we use to send a man to the moon, and that has inspired the likes of Plato, Einstein and Stephen Hawking, has something in common with what a rat is doing when it's out hunting for food," he said. "I find that deeply moving."

A version of the test can be found online at <www.nytimes.com/interactive/2008/09/15/science/20080915_NUMBER_SENSE_GRAPHIC.html>

Citation: Angier, Natalie. "Gut Instinct's Surprising Role in Math," New York Times, September 15, 2008, retrieved September 16, 2008 from: </www.nytimes.com/2008/09/16/science/16angi.html?8dpc>

How Did You Solve That Problem? Letting Students Do Math Their Way Promotes Equity

Quotes and excerpts appearing in this article are from:

Imm, Kara Louise, Stylianou, Despina A., and Chae, Nabin (2008). "Student Representations as the Center: Promoting Classroom Equity," *Mathematics Teaching in the Middle School,* NCTM, Reston, VA, Vol 13, No. 8, April 2008, pp.458-463.

In diverse classrooms, like those found in the inner city, teachers are always conscious of creating a classroom environment that welcomes everyone and makes each student feel important. A New York City teacher shared "The Playground Problem" (see next page) with her class and allowed students to work in pairs to solve the problem and then share the representations of their work. Along the way, teachers discover the power of sharing student-generated representations, the power to facilitate mathematical growth while fostering equity.

> By relocating student-generated representations to the center of the instruction, the nature of how students experience mathematics changes dramatically.

In their article, "Student Representations as the Center: Promoting Classroom Equity," the teacher and two other researchers note that:

Even at a young age, students come to school with their own, often culturally influenced, valid representations (Lave, 1998). Because those representations have been crafted, interpreted and modified by the students themselves, they become vital to classroom instruction. To dismiss what students bring naturally to the classroom reduces mathematics to a one-way transaction between teacher as expert and student as novice, confirming the notion that a student's own thinking and all that he or she brings to mathematics is marginal at best. By relocating student-generated representations to the center of the instruction, the nature of how students experience mathematics changes dramatically. It reconsiders mathematics as a vibrant dialogue among different but equally valued thinkers. This deliberate approach to the teaching of mathematics, we believe, becomes vital if we are serious about creating greater equity for our students. P.459

As the class discusses various representations of the problem, the authors explain, "students begin to use Charlene and Joey's model and refer to it as 'Charlene and Joey's strategy.' They see this an example of "the principle of 'authority in which the learning environment 'authorizes' students to attempt to solve mathematical problems" and reflects the fact they fellow students credit them as "authorities in the discipline," a notion they see "not only (as) an issue of building confidence in students but of changing students' perceptions of mathematics with classroom community." "Making mathematics a part of the culture of the community," they assert, leads to an acknowledgement "that

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The Connection Between Teacher and Student Performance

…the effect that elementary school teachers have on student achievement gains is considerable — 10 to 15 percent in a single year, particularly in mathematics, and even more when viewed cumulatively...

—Quote from *No Common Denominator: The Preparation of Elementary School Teachers in Mathematics* by America's Education Schools, National Council on School Quality, June (2008)

How Did You...
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a particular representation was not imposed by an outside authority but was created by a mathematician among us." (p. 461) Students begin to perceive of themselves as mathematicians capable of solving and representing problems. Math does not happen to them, they are part of the culture of mathematicians.

The authors conclude that attaining equity "involves validating the knowledge that (students) bring to classroom settings, which may include cultural, linguistic, and informal ways of knowing in addition to formal, or academic, learning…." They maintain "appropriate use of student-generated representations in a classroom can be a powerful tool, not only with respect to students' mathematical growth, but also in our perennial struggle to attain equity in mathematics classes." (p. 462) These are lofty goals worthy of all teachers.

Imm, Styliano, and Chae on 'Representation'

"The NCTM's Standards (2000) suggest that a representation is not only a product (a pictue, a graph, a number, or a symbolic expression) but also a process, a vehicle for developing an understanding of a mathematical concept and communicating about mathematics. To serve as vehicle in learning and communication, however, a representation must be personally relevant and meaningful to a student."

The Playground Problem

Two communities in Brooklyn, Carroll Gardens and Flatbush, each gather to make plans for an empty lot in their neighborhood. The lots are identical in size, measuring 50 yards x 100 yards. In Carroll Gardens, the community group decides to allocate 3/4 of the empty lot to playground and cover 2/5 of this playground with blacktop. The Flatbush neighborhood will devote 2/5 of the lot to playground, and 3/4 of the playground will be covered in blacktop. In which park is the blacktop area greater? Show your solution and be prepared to justify your solution.